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Learning Outcomes
Students will:
- Ask questions that could be explored using mathematics in relation to MYP content.
- Analyse a mathematical or real-life situation, solving problems using technology where applicable.
- Use mathematical terminology and notation, algebraic symbols, diagrams, text and tables to communicate mathematical ideas.
- Identify relationships and the strengths and weaknesses of different strategies and solutions, giving reasons.
- Link mathematical ideas and makes connections with and generalisations about existing knowledge and understanding in relation to MYP content.

Areas of Interaction
Approaches to Learning, Homo Faber
CHAPTER 2 WORKING MATHEMATICALLY: APPLYING STRATEGIES

2:01 | Direct Computation

Fun Spot 2:01 | Can you play ‘31’?

This is a game for two players.
- Start with 31 counters.
- The first player chooses a number from 1 to 6 and removes this number of counters from the pile.
- The second player also chooses a number from 1 to 6 and removes that number of counters.
- The players continue in turn to remove up to 6 counters until all counters have been removed.
- The player forced to take the last counter is the loser.

A winning strategy
- Play the game with a partner.
- Can you work out a winning strategy? (The person who goes first should always win.)
- Look to the answers for an explanation of the winning strategy.

In your lifetime you will meet thousands of real problems waiting to be solved:
‘What is the quickest way from Heathrow to Northfields?’ ‘How can I rearrange my working day so that I can attend a meeting at Brooklyn?’ ‘How can I build a skateboard?’ ‘How should I arrange the dinner party?’

We can see life as an exciting adventure or as a burden. We each have our own ways of solving problems. In this chapter we will examine some exciting and challenging strategies to help us solve unusual problems.

Steps for solving problems
Step 1 Read the question carefully.
Step 2 Decide what you are asked to find.
Step 3 Look for information that might be helpful.
Step 4 Decide on the method you will use.
Step 5 Set out your solution clearly, using solution sentences.
Step 6 Make sure that your answer makes sense.

Always try to show clearly how you have solved the problem so that others can understand and investigate the problem you have attempted.

Setting out
Find = method
= solution
Answer (in words):

Working if needed:
worked example 1

Anna was told that she would be given €15 allowance each week if she was helpful around the home. She decided to bank €2.75 each week and do her best to help around the home. How much would she expect to bank in nine weeks?

Solution
Considering the problem:
(a) What are we asked to find?
What Anna would expect to bank in 9 weeks.
(b) What information is given?
What is needed?
Given: Allowance is €15, banks €2.75 per week.
Needed: Banks €2.75 per week.
(c) What method should we use?
Multiply the amount banked weekly by the number of weeks.
(d) What solution sentences will we use?
Amount banked = 9 × €2.75.

\[
\text{Amount banked} = 9 \times \€2.75 = \€24.75
\]

In nine weeks Anna banks €24.75.

worked example 2

Giulio purchased a table for 132 pesos and a radio for 86 pesos. After spending 26.50 pesos on repairs, he sold the two items to a customer for 349.95 pesos. How much profit did he make altogether on the two items?

Solution
Considering the problem:
(a) What are we asked to find?
The profit made altogether on two items.
(b) What are we given?
Cost of items and repairs and selling price.
(c) What method should we use?
Find total cost first. Then subtract this from the selling price of the items.
(d) What solution sentences will we use?
(i) Total cost = cost 1 + cost 2 + repairs.
(ii) Profit on the items = selling price − total cost.

Note: This problem could have been done as a one-step problem:
Profit on the items = selling price − (total costs)
Profit on the items = \( \$349.95 - (132 + 86 + 26.50) \)
Profit on the items = \( \$105.45 \)
∴ Giulio made \( \$105.45 \) profit on the two items.
Exercise 2:01

After comparing prices at three stores, Peter found that the cheapest price for mixed canary seed was $3.45 for 5 kg. He bought 15 kg of seed. How much did he pay for each kilogram of seed and how much did he pay altogether?

a How much seed did Peter buy?

b What was the cost of 5 kg of seed?

c To how many stores did Peter go to compare prices?

d What two things are we asked to find?

e Solve the problem, setting out your solution as in the examples.

At Greentrees bookshop I bought 3 of the Narnia books at £3.30 each, a Bible at £14.95 and 10 bookmarks at 35 pence each. How much did I spend altogether?

a What am I asked to find?

b How many bookmarks were bought?

c What was the total cost of the bookmarks?

d How many items were bought altogether?

e Solve the problem, setting out your answer as shown in the example.

Solve the following problems, showing your setting out as in the examples.

3 How many eight-page books can be made from 618 pages?

4 Jennifer bought a television set for $875.75 and a DVD player for $963.40. If she had saved $2000 to buy these two items, how much would she have left?

5 Mr and Mrs Lucre were returning home after spending a day at the beach. After travelling for three hours at 60 kilometres per hour they were still 11 kilometres from home. How far from home were they when they began the trip home?

6 In May, Alan bought 55 different animal magazines for 25 cents each. In August he bought 47 others, all different from those purchased already. If the whole set contains 120 different magazines, how many magazines are missing from Alan’s collection?

7 Luke has 40 toy cars and 100 marbles. If he shares these with his four sisters, what would his oldest sister be given?

8 Kylie cut an apple into 12 equal parts. If she ate \(\frac{1}{4}\) of the apple, how many parts would remain?

9 The total number of votes cast in three electorates was 654 816. If in one of these electorates 211 302 votes were cast and in another 284 965 votes were cast, how many votes were cast in the third electorate?

10 Marie and Adam went to a Chinese restaurant to celebrate their engagement. The dishes they liked were: Sweet and Sour Chicken at $8.50, Steamed Duck with Sweet Corn at $10.95 and Braised Lobster with Seafood at $20.95. Dessert was $3.75 per serving and tea was $1.25 per person. What would be the total cost if Marie chose the lobster, Adam chose the duck and they each had dessert and tea?
How many ♥♦♣♠ symbols are found on a pack of playing cards?

Here’s the question: How do I do it sir?

Just choose a strategy and work with it. But what if it fails?

If it fails, choose another strategy. A problem is a challenge, not a barrier.

Look for patterns.

Restate the problem in your own words. How many of those funny dots are on the 52 cards?

Eliminate possibilities. Make a drawing, diagram or model.

Identify sub-goals. Solve a simpler problem. Make a list, chart, table or tally.

Let’s find the number of dots on one suit.

Draw a graph.

This doesn’t seem to be helping.

Does your answer make sense?

ESTIMATE 600

Does the Jack, Queen and King have any hidden dots?

Work backwards. Act out the problem.

In this case, these two strategies don’t seem to help.

But what about the Jack, Queen and King?

I guess that the 2 of spades has 4 dots.

Sample pages
Working mathematically

Throughout this course you will apply the strategies treated in this chapter.

Along with applying strategies you will be developing your skills in questioning, communicating, reasoning and reflecting. These are all ways in which we work mathematically.

You now need to practise using these strategies so that you can develop skill in choosing which strategies to use when you are solving a problem. Very often you can use more than one strategy to get an answer, but before you begin you should estimate the answer.

To estimate, ask:

• Will the answer be big or small?
• How big? How small?
• Will the answer be a whole number?
• Does my estimate make sense? Is it reasonable?

**Reading Mathematics 2:02 | A standard pack of cards**

A standard pack of cards has 4 ‘suits’: hearts, diamonds, clubs and spades. Hearts and diamonds are red. Clubs and spades are black.

![Hearts](image1)

![Diamonds](image2)

![Clubs](image3)

![Spades](image4)

In each suit there are 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. All the hearts are shown below.

![Hearts Cards](image5)

Since there are 4 suits with 13 cards in each suit, the number of cards in a standard pack is 52. (In some games a Joker is also used.)
Another name for trial and error is guess, check and refine. Trial and error is a useful problem-solving strategy. Its only drawback is that for some problems it is too slow and time-consuming.

**worked example 1**

Osu is five years older than Erna. Twice Osu’s age plus three times Erna’s age is equal to 25. What are the ages of the two girls?

**Solution**

Osu is 5 years older than Erna.

**Guess 1:** Osu is 10, Erna is 5.

**Check 1:** \( (2 \times 10) + (3 \times 5) = 20 + 15 = 35 \)

This is too large.

**Guess 2:** Osu is 8, Erna is 3

**Check 2:** \( (2 \times 8) + (3 \times 3) = 16 + 9 = 25 \)

This is correct.

∴ Osu is 8 years old and Erna is 3 years old.

**worked example 2**

Sandra’s cycle repair shop fixes normal bicycles and tricycles. Sandra had to assemble a number of bicycles and tricycles. She counted 15 handlebars and 36 wheels among the parts she had to put together. How many of each type of cycle were to be assembled?

**Solution**

Since there are 15 handlebars, there must be 15 cycles.

**Guess 1:** 7 bicycles and 8 tricycles.

**Check 1:** Number of wheels = \( (7 \times 2) + (8 \times 3) \)

\[ = 14 + 24 \]

\[ = 38 \]

This is too big, so try fewer tricycles.

**Guess 2:** 9 bicycles and 6 tricycles.

**Check 2:** Number of wheels = \( (9 \times 2) + (6 \times 3) \)

\[ = 18 + 18 \]

\[ = 36 \]

This is the correct number of wheels.

∴ There are 9 bicycles and 6 tricycles to be assembled.

**Exercise 2:03**

1. Mary is six years older than Jean. The sum of their ages is 20. How old is Mary?

2. Su-Lin is four years older than Mark. Twice Su-Lin’s age plus five times Mark’s age is equal to 64. What are the ages of Su-Lin and Mark?

3. Derek was 12 years old when his sister was born. Now the sum of their ages is 58. How old is Derek now?
4. The sum of two numbers is 140 and their difference is 6. Find the two numbers.

5. The product of two numbers is 72 and their sum is 27. What are the numbers?

6. The product of two numbers is 741 and their sum is 70. What are the numbers?

7. A farmer has chickens and cows. If there are 18 heads and 52 feet on these altogether, how many cows are there?

8. Diane keeps ants and spiders as pets. She counted the legs and heads of these pets and found that there were 136 legs and 21 heads. How many spiders were there?

9. From the digits 3, 4 and 5, two numbers are formed, a two-digit number and a one-digit number. What must the two numbers be if:
   a. the product is to be as big as possible?
   b. the product is to be as small as possible?
   c. the product is to be 170?

10. Two two-digit numbers are made from four of the digits 1, 2, 3, 4 and 5, with no digit being used more than once. What numbers would produce:
    a. the smallest product?
    b. the largest product?
    c. a product closest to 700?

11. Roland sold bananas for 15 cents and apples for 20 cents. He received $3.50 for a mixture of bananas and apples. If twice as many bananas as apples were sold, how many apples were sold?

12. Kai-Lin answered all 25 questions on a test in which she received 4 marks for each correct answer but lost one mark for each incorrect answer. If her mark was 65, how many questions did she get right?

13. Find the counting number that would replace the square to make the sentence true.
   \[16 \times \square - 72 = 56\]

14. Louie, Verity and their three children went to the movies. The children were charged half price. If the total cost of the tickets was $52.50, how much did each adult ticket cost?

15. Four non-zero digits are placed in these four squares to make two two-digit numbers across, and two two-digit numbers down. If the sum of these four two-digit numbers is 77, what are they?

16. Place the numbers below into two groups of four, so that the sum of one group is as close as possible to the sum of the other group.
   \{9, 10, 18, 21, 25, 42, 49, 51\}

17. Leonie is 35 years older than her daughter Louise but 3 years younger than her husband Max. If the sum of the three ages is 94, how old is Leonie?
Kalmna, April and Lindsay scored 196 marks altogether on a test. If April scored 15 less than Kalmna and 7 less than Lindsay, how many marks did April score?

I think of a counting number, add one, multiply the answer by itself, then take away twice the number I first thought of. If my answer is 26, what was the number I first thought of?

Twice Alana’s age plus three times Luke’s age is equal to 25. What are the possible totals of their ages?

2:04 | Making a Drawing, Diagram or Model

When you draw a picture to represent a problem, you can often see what must be done. It is said that ‘a picture is worth a thousand words.’

**worked example 1**

4 km of fencing encloses a square paddock of area 100 hectares. If 8 km of fencing was used to enclose a square paddock, how many hectares would be enclosed?

**Solution**
A diagram or drawing allows us to see the problem more clearly.

Even though the length of fencing has only been doubled, the area has become 4 times as great.

∴ 400 hectares would be enclosed by 8 km of fencing.

**worked example 2**

Rhonda and Alan married and had five children. Each child married and had three children. Assuming no-one has died, how many people are now in this extended family altogether?

**Solution**
Unless you draw a diagram you may miss some of the people.

For example, you might forget to include the people who married the five children.

From the diagram you can see that there are 27 people in the family.
1. 4 km of fencing encloses a square paddock of area 100 hectares. If 16 km of fencing was used to enclose a square paddock, how many hectares would be enclosed?

2. Rachel had one pair of guinea pigs. These had four baby guinea pigs, all female. Within the following six months, each female guinea pig had produced five baby guinea pigs. If no guinea pigs died, how many did Rachel have at the end of that six-month period?

3. Each child in the Parker family had at least two brothers and one sister. What is the least number of children in the family?

4. a. I want to make a line of posts 8 m long. If I place the posts 1 m apart, how many posts will I need?
   b. If I were to use 24 posts and place them 2 m apart, how far would they stretch?
   c. A line of posts 112 m long is made with posts equally spaced. If 8 posts are used, what is the distance between posts?

5. How many counting numbers are less than 3 units from the number 4?

6. a. Eight equally spaced posts are used as uprights in a square pen. How many posts would be placed on each side of the pen?
   b. A square pen is made using four posts on each side. How many posts were used?

7. If it takes Dave 6 minutes to cut a pipe into two pieces, how long would it take him to cut it into five pieces?

8. It took Luke $1\frac{1}{2}$ seconds to join two pieces of Lego. How long would it take him to join seven pieces into one row of Lego?

9. A triangle and a rectangle are drawn on a page so that they overlap as shown. Inside these figures are placed the digits from 0 to 9. If there are six digits in the triangle and eight digits in the rectangle, how many digits are in the part that belongs to both figures?
a Nineteen students were waiting in line to be chosen to play in a game. Mr Archbold chose the first student in line and every third one after that. How many students were chosen?
b 99 Roman soldiers who fled from battle were to be punished. The group was lined up and decimated (every 10th one was killed). How many were killed?
c How many of the 99 soldiers would be killed if the second soldier was killed and every tenth soldier after that?

A doctor prescribes tablets for June to relieve her pain. They must be taken at least an hour apart and no more than four tablets may be taken in any six-hour period. What is the largest number of tablets she may take in 14 1/2 hours?

Two proofreaders are reading the same work. One finds 50 mistakes, the other 63 mistakes. If 40 of the mistakes were found by both readers, how many mistakes were found?

At the start of each week, May put $400 into her cheque account. During the week she would then withdraw $300. Assuming that there are no bank or government charges, in which week would she have $1000 in the bank for the first time?

The 30 students in 6E study both History and Geography. All students like at least one of these two subjects. If 18 students like History and 7 students like both subjects, how many students like Geography?

Jock decided to swim 10 km up his local river without leaving the river. He swam 2 km every hour then floated on his back for 10 minutes. Each time he floated, the current dragged him back 1 km. How long did it take Jock to complete the swim?

Investigation 2:04 | Catching the crooks

Please use the Assessment Grid on the following page to help you understand what is required for this Investigation.

Prince Charming was suspicious of his crown makers. He suspected that they were not using all the gold that he provided to make his crowns. He decided to test the crown makers. He ordered three different crown makers to make identical crowns for him. He made each crown maker think that only one crown was being made.

How could he discover if the crown makers were cheating him? What problems might occur to make his task difficult?
Assessment Grid for Investigation 2:04 | Catching the crooks

The following is a sample assessment grid for this investigation. You should carefully read the criteria before beginning the investigation so that you know what is required.

<table>
<thead>
<tr>
<th>Assessment Criteria (B, C, D)</th>
<th>Achieved ✓</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Criterion B</strong></td>
<td></td>
</tr>
<tr>
<td>a) No systematic approach has been used.</td>
<td>1</td>
</tr>
<tr>
<td>b) An organised approach has been used with limited success.</td>
<td>3</td>
</tr>
<tr>
<td>c) A systematic approach has been used successfully, but is not well explained.</td>
<td>5</td>
</tr>
<tr>
<td>d) The correct result has been obtained by using an organised approach which is reasonably well explained.</td>
<td>7</td>
</tr>
<tr>
<td>e) The correct result has been obtained by using an organised approach which is well explained.</td>
<td>9</td>
</tr>
<tr>
<td><strong>Criterion C</strong></td>
<td></td>
</tr>
<tr>
<td>a) No working out or explanation has been given.</td>
<td>1</td>
</tr>
<tr>
<td>b) There is some structure to the work making it possible to follow the steps undertaken.</td>
<td>3</td>
</tr>
<tr>
<td>c) Results and the steps taken to get them are well explained. The work is well structured.</td>
<td>5</td>
</tr>
<tr>
<td><strong>Criterion D</strong></td>
<td></td>
</tr>
<tr>
<td>a) There has been some attempt to explain the method used and to check if the result works.</td>
<td>1</td>
</tr>
<tr>
<td>b) The method and the majority of the processes used are justified and the reliability of the findings have been checked with some success.</td>
<td>3</td>
</tr>
<tr>
<td>c) A reasoned explanation of the method used and the reliability of the findings have been given.</td>
<td>5</td>
</tr>
<tr>
<td>d) Problems that could be encountered are discussed and possible solutions have been provided.</td>
<td>7</td>
</tr>
</tbody>
</table>
2:05 | Make a List, Chart, Table or Tally

You will often need to make a complete list to solve a problem. It is very important that you select the best way to make your list.

worked example 1

Alana has to list the names of three universities in the order she prefers. The universities she likes are National University of Singapore (NUS), the University of Tokyo (UTK) and Hong Kong University (HKU). How many different ways are there to list these universities in order?

Solution

The way you make your list is very important. It should help you not to leave out any part of the list. We will use abbreviations NUS, UTK and HKU.

Note: 
(1) Once the first choice has been made there are two left to choose from in making choice 2.
(2) Once the first two choices have been made only one choice is left for choice 3.

<table>
<thead>
<tr>
<th>Choice 1</th>
<th>Choice 2</th>
<th>Choice 3</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUS</td>
<td>UTK</td>
<td>HKU</td>
<td>NUS</td>
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<tr>
<td>UTK</td>
<td>HKU</td>
<td>NUS</td>
<td>UTK</td>
</tr>
</tbody>
</table>

worked example 2

The numerals 1 to 10 are written on ten separate cards, one on each card.

(a) How many pairs of cards have a sum of 8?
(b) How many groups of three cards are there that have a sum of 20?

Solution

(a) The numbers must add to give 8.

We can see that three pairs of cards have a sum of 8.
a A ‘Carols by Candlelight’ program order must be decided. The items to be put in order are Carols (C), Entertainers (E) and Sermon (S). If the sermon must come either 2nd or 3rd of the items, in how many ways can the program be arranged?

b It is decided that the entertainers Julie Anthony and Kamahl should be considered as two separate items, while the sermon must be placed 3rd or 4th on the program. How many ways are there of arranging the items Carols (C), Anthony (A), Kamahl (K) and Sermon (S)?

From five girls, two must be chosen to represent our club at tennis. How many different pairs could be chosen? Call the girls A, B, C, D and E.

(Note: AB is the same pair as BA.)

The numerals 1 to 7 are written on seven separate cards, one on each card.
a How many pairs of cards add up to 8?
b How many groups of three cards are there that have a sum of 10?

How many numerals less than 100 contain the digit 5?

a If each digit is used only once, how many two-digit numbers can be made using the digits 1, 2, 3, 4? (Note: 21 is different from 12.)
b If each digit can be used more than once, how many two-digit numbers can be made using the digits 1, 2, 3, 4?
c If each digit can be used only once, how many two-digit numbers can be made using the digits 0, 1, 2, 3?

Five people were waiting for an elevator. When the elevator arrived, there was room for only three people. How many different groups of three people could get in the elevator? (Call the people A, B, C, D, E.)
David and Paul own a sailing boat that carries only two people. Four friends, Luke, Naomi, Rachel and Alana, would like to ride in the boat too. Since only David and Paul know how to sail the boat, at least one of these two must go out in the boat whenever it sails. How many pairs could be chosen to go out in the sailing boat?

How many different ways can the letters A, E, B and T be arranged so that the T is the 3rd or 4th letter? (AETB and AEBT are two ways.)

How many of the patterns in (a) are English words?

Maurice saw the registration plate of a car involved in a bank robbery. He told the police that the plate had three letters followed by three numbers. He knew that the first letter was E and the other two letters were Z and A but he was unsure of the order of the Z and A. The three numbers were 2, 3 and 8 but he did not know their order. How many registration plate would fit this description?

A school captain and vice-captain are to be elected from seven candidates. How many different results can occur?

Two dice are thrown. One has the digits 1 to 6 written on the faces while the other has the letters A, B, C, D, E and F on the six faces (one on each face of course).

How many results could contain an A or E (a vowel)?

How many different results are possible?

If every letter of the alphabet could be followed by a one-digit number, how many of these pairs could be formed?

I own 3 ties, 4 shirts and 2 belts. How many different ways could I wear a tie, a shirt and a belt. (Hint: Call the ties T₁, T₂, T₃, the shirts S₁, S₂, S₃, S₄, and the belts B₁ and B₂.)

A set of dominoes is a collection of rectangular pieces that have two parts. On each part, one of the numbers 0 to 6 is represented by dots. All possible combinations of numbers are made but no two dominoes are alike.

How many dominoes are there in a set?

What is the sum of the numbers shown on a set of dominoes?

A number of cards can be shared between 4 people exactly, but when shared between 5 people there are two cards over. When shared between 6 people there are four cards over. If there are fewer than 80 cards, how many cards are there?

How many different amounts can be made up using three of the six Australian coins: 5c, 10c, 20c, 50c, $1, $2?
2:06 | Eliminating Possibilities

Sometimes it’s possible to find the right answer by eliminating answers that are wrong.

worked example

Three students studying in Vietnam who were born in different countries have last names Sze, Kalra and Poon. Their first names are Sundeep, Christopher and Andy but not necessarily in that order. Sze was born in Indonesia, Poon has never been to the Philippines, Andy was born in Malaysia and Sundeep was born in the Philippines. What is the full name of each student?

Solution

We use tables to eliminate as many possibilities as we can. We hope that what is left will be the answer.

- As Sze was born in Indonesia we can put √ √ × for Sze.
- Poon has never been to the Philippines so for Poon cross the Philippines box. ×
- Andy was born in Malaysia so put √ × × for Andy.
- Sundeep was born in the Philippines so put × √ √ for Sundeep.

Now as each person was born in a different country, the fact that neither Sze nor Poon was born in the Philippines means that Kalra must have been. Tick the box Philippines/Kalra. × In this way the other boxes can be filled in.

From the table we can see that:

- Sze and Chris were born in Indonesia.
- Chris Sze was born in Indonesia.
- Kalra and Sundeep were born in the Philippines.
- Sundeep Kalra was born in the Philippines

Exercise 2:06

1. Find the number between 1 and 10 that is not even, and when divided by 3 gives a remainder of 1.

2. Find the number less than 50 that has the sum of its digits equal to 6 and the difference of its digits equal to 4.

3. Find the number that is between 0 and 120, is even, is a multiple of 5 and is the result of multiplying one of the counting numbers by itself.

A multiple of 5 is a number that can be exactly divided by 5, eg 5, 10, 15, 20 . . .
4 Find the number between 1 and 50 that is not a multiple of 2 or 3 and whose two digits differ by 4.

5 If ties sell for $8.60, $9.20, $10.80 and $12.40, which of these could be the cost of 6 ties?
   a $51.40    b $74.80    c $63.30    d $62.60

6 Vinh and his wife Thanh owned a grocery store in Hanoi. They received ten sample tins, all of which had a net mass of 225 g, 400 g, 425 g or 450 g. Which of the following could be the total net mass of the ten sample tins?
   a 4600 g    b 3830 g    c 3625 g    d 1850 g

7 Three children, Jennifer, Kylie and Bronwyn, are friends. Their surnames are Conway, Collison and Wilkes, but not necessarily in that order. Jennifer lives in a red house, Bronwyn lives in a yellow house, Miss Conway lives in a blue house and Miss Wilkes does not live in a red house. What are the full names of each of the girls?

8 Which fruit did I grow?
   My fruit:
   • is not red    • is not narrow    • is not green

   A  B  C  D  E  F  G  H  

   Work out the least and greatest possible cost of six ties.
For some problems, working backwards gives a solution.

**worked example**

Mary is 35 years younger than Tom. Fred is half the age of Mary. Judy is 17 years older than Fred. If Judy is 35, how old is Tom?

**Solution**

When working backwards we often use opposite operations.

We know that Judy is 35.

We know that Judy is 17 years older than Fred.

So Fred's age is $35 - 17$, which is 18.

We know that Fred is half the age of Mary.

So Mary's age is $18 \times 2$, which is 36.

We know that Mary is 35 years younger than Tom. So Tom's age is $36 + 35$, which is 71.

∴ Tom is 71 years old.

**Exercise 2:07**

1. Neil is 14 years younger than Hayley. Lisa is 26 years older than Neil. If Lisa is 37, how old is Hayley?

2. Four people weighed themselves. Young Louise was 17 kg lighter than Adele, Gavin was twice as heavy as Louise, and Andrew was 9 kg heavier than Gavin. If Andrew’s weight was 73 kg, what was Adele's weight?

3. My age is 60. I was married 23 years ago and graduated 5 years after that. How old was I when I graduated?

4. Hungry Harrie has $48 now but during the last week he has bought ten hamburgers at $1.85 each and eight milkshakes at 90 cents each. He also earned $21.50 mowing lawns during the last week. How much money did he have one week ago?

5. I think of a number, multiply it by 4, then subtract 8 from that answer. I am left with 72. What was the number I first thought of?

6. When Rodney arrived in Belgium he had some money to help him get started. In the first three years this amount grew to 10 times its original size. With this money he started a plant nursery and with a lot of hard work built up his personal wealth by a further €98 000 in a few years. If his personal wealth was now €148 000, how much did he have when he arrived in Belgium?
Sometimes it’s possible to act out the problem to find the solution.

**worked example**

Three circular discs, which have holes through their centres, are to be moved from the right peg to the left peg. The discs are to be moved one at a time and no disc can be placed on top of a smaller one. What is the least number of moves needed to move the discs to the left peg?

**Solution**

You could cut out 3 paper circles and act out this problem yourself.

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

The least number of moves needed to move the discs to the left peg is 7.

**Exercise 2:08**

Three circular discs, which have holes through their centres, are to be moved to the left peg. The discs are to be moved one at a time and no disc can be placed on top of a smaller one. What is the least number of moves needed to move the discs to the left peg in each of the following cases?

a

b

c
2 An ant wishes to cross from one side of a draughts board to the other. He feels that he must always keep a black square on his immediate left, and always walks along the lines on the board. If the squares on the board have sides of length 3 cm, what is the least distance he must travel?

3 A mouse sits on a black square that is on one edge of the board. She decides only to step on black squares, and only to move to other black squares that are touching the black square on which she sits. Onto how many squares must she move to reach each of the other three edges and return to her starting point again?

4 The knight (a chess piece that looks like the head of a horse) moves on the board by going two squares in one direction and then one to the side (or one square in one direction then two to the side). This is counted as one move. What is the least number of moves necessary to get the knight from one corner to the opposite corner?

5 If the mouse steps as in question 3 and starts where she is on the board above, what is the greatest number of black squares that she can step onto in one trip if she must not tread on the same square twice?

6 Walking to church, dressed in their new clothes, Mr and Mrs Ritchie and their two children Andrew and Vanessa came to a very muddy part of the road. Fortunately Mr Ritchie's donkey was grazing nearby. It could, however, only carry one adult or two children at the one time. Explain how the whole family got over the mud using the donkey.

---

**Challenge 2:08 | Backtracking**

\( \square + 10 \) means \( \square \) is changed into 84 by adding ten.

Find the numbers represented by the frames in each of the examples below.

1. \( \square + 10 \rightarrow 84 \)
2. \( \square + 8 \rightarrow 37 \)
3. \( \square - 2 \rightarrow 106 \)
4. \( \square - 3 \rightarrow 17 \)
5. \( \square + 2 \rightarrow \quad \square + 10 \rightarrow 36 \)
6. \( \square + 3 \rightarrow \quad \square - 8 \rightarrow 1 \)
7. \( \square - 7 \rightarrow \quad \square + 6 \rightarrow 67 \)
8. \( \square + 3 \rightarrow \quad \square + 5 \rightarrow \quad \square - 9 \rightarrow 11 \)
Looking for Patterns

If you can find a pattern, it may help you solve the problem.

worked examples

Look for patterns that will help you find the answer to each of these.

a  \( 7 - 6 + 7 - 6 + 7 - 6 + 7 - 6 + 7 - 6 + 7 - 6 + 7 - 6 + 7 \)

b  \( 9 + 5 + 2 + 9 + 5 + 2 + 9 + 5 + 2 + 9 + 5 + 2 \)

c  \( (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) + (9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) \)

Solutions

a  By grouping the numbers in pairs, the answer is quickly found.
\[
(7 - 6) + (7 - 6) + (7 - 6) + (7 - 6) + (7 - 6) + (7 - 6) + (7 - 6) + (7 - 6) + 7 \\
= (ten \text{ lots of } 1) + 7 \\
= 17
\]

b  Here we can arrange the numbers in groups of three.
\[
(9 + 5 + 2) + (9 + 5 + 2) + (9 + 5 + 2) + (9 + 5 + 2) + (9 + 5 + 2) + (9 + 5 + 2) \\
= 6 \text{ lots of } 16 \\
= 96
\]

c  If we add the first number in each group, we get 10. The second numbers also have a sum of 10, and this pattern continues.
\[
(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) + (9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) \\
\text{Adding } 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10
\]
Even without showing working we could have seen that there were 9 lots of 10.
\[
\therefore \text{ The answer is } 90.
\]

We can use this idea to find other sums.
To add the counting numbers from 1 to 200, we could use them twice and then divide that answer by 2,

\[
(1 + 2 + 3 + 4 + 5 + 6 + \ldots + 199 + 200) + (200 + 199 + 198 + 197 + 196 + 195 + \ldots + 2 + 1) \\
= 201 + 201 + 201 + 201 + 201 + \ldots + 201 + 201
\]
Altogether, twice the sum gives us 200 lots of 201.
\[
\therefore \text{ The sum of the counting numbers from 1 to 200 } = \frac{1}{2} \text{ of } (200 \times 201) \\
= 100 \times 201 \\
= 20100
\]

Many problems involving patterns will be met in Chapter 8, ‘Patterns and Algebra’.
Exercise 2:09

1 Look for patterns which will help you find the answer to each of these.
   a) $9 - 7 + 9 - 7 + 9 - 7 + 9 - 7 + 9 - 7 + 9 - 7 + 9$
   b) $1 + 2 + 3 + 4 + 1 + 2 + 3 + 4 + 1 + 2 + 3 + 4$
   c) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \frac{1}{10}$
   e) $197 - 187 + 177 - 167 + 157 - 147 + 137 - 127 + 117 - 107 + 97 - 87 + 77 - 67$

2 Use the method given before this exercise to find the sum of the counting numbers from:
   a) 1 to 400
   b) 1 to 1000
   c) 1 to 100
   d) 1 to 50

2:10 Solving a Simpler Problem

If a problem seems complicated, try solving a simpler problem or see if you can solve the problem using easier numbers.

worked example

A machine is producing a roll of grid paper 2 cm wide. Within the grid there are squares with sides 1 cm in length and squares with sides 2 cm in length.

How many squares can be found if the grid has a length of 100 cm?

Solution

To solve this problem would take a long time so we solve simpler problems and look for a pattern. Try smaller lengths.

<table>
<thead>
<tr>
<th>Length</th>
<th>Number of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length</th>
<th>Number of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 cm</td>
<td>4 + 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length</th>
<th>Number of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 cm</td>
<td>6 + 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length</th>
<th>Number of squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 cm</td>
<td>8 + 3</td>
</tr>
</tbody>
</table>

continued
The hare challenges the tortoise to a race. The hare can travel 10 m for every 1 m travelled by the tortoise. If they continue to travel at this rate, by how much will the hare beat the tortoise if the race is over a distance of:

- **a** 10 m?
- **b** 20 m?
- **c** 100 m?
- **d** 1000 m?
- **e** 2 km?

How much start could the hare in question 1 give to the tortoise in a race over 50 m and still expect to beat the tortoise?

Blackbeard the pirate was following an unusual treasure map: 'Jump from Dead Man’s Rock onto the wet sand and then take 54 steps along the beach.'

- **a** If he obeyed the instructions exactly, how many shoeprints would he have made in the sand?
- **b** If he only had one good leg and a ‘pegleg’ on the other, how many shoeprints would he have made?

How many diagonals can be drawn from one corner of:

- **a** a square?
- **b** a pentagon (5 sides)?
- **c** a hexagon (6 sides)?
- **d** a heptagon (7 sides)?
- **e** a figure on a flat surface that has 100 sides?

### Exercise 2:10

<table>
<thead>
<tr>
<th>Length (cm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of squares</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>299</td>
</tr>
</tbody>
</table>

∴ 299 squares can be found in 100 cm of grid paper.

An octagon has five diagonals drawn from one corner.
Ten prospectors are working in a gold mine. Every prospector fills a small bag of gold each month. After the first month one prospector disappears, taking with him all the gold dug up in that month. The same thing happens in each of the following months until only one prospector is left. How many bags of gold were stolen?

If there had been five honest prospectors among those in question 5, how many bags of gold would have been stolen?

Looking again at the example before this set, write down how many lines of length 1 cm are needed to draw a grid of length:

- a. 1 cm
- b. 2 cm
- c. 3 cm
- d. 4 cm
- e. 5 cm

In question 7, how many extra 1 cm lengths were needed for each centimetre increase in length? Use this information to find the number of centimetre parts needed to draw a grid of length 100 cm.

At a church meeting, each person who arrived shook the hand of every other person present. Upon arrival, how many hands would be shaken by:

- a. the first person?
- b. the second person?
- c. the third person?
- d. the fourth person?
- e. the fifth person
- f. the sixth person?
- g. the seventh person?
- h. the eighth person?

If every person at the meeting in question 9 shakes hands with every other person there, how many handshakes will there be altogether if:

- a. 2 people are present?
- b. 3 people are present?
- c. 4 people are present?
- d. 5 people are present?
- e. 6 people are present?
- f. 7 people are present?

Use the answers to question 10 to complete this table.

<table>
<thead>
<tr>
<th>Number of people</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of handshakes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Can you see a pattern in the table? What is it?

Extend this pattern to find the number of handshakes needed if 20 people were present.

Note: Other methods, like drawing a picture, making a list or acting it out, can be used to help solve this problem.
Let’s post a parcel

The amount to be paid for posting a parcel will depend on the destination (where it is going) and the weight of the parcel.

This table assumes that the parcel is being sent from a zip code that is in Zone 1.

Table 1 — Parcel charging zones (Destination)

<table>
<thead>
<tr>
<th>Zip code</th>
<th>Zone 1</th>
<th>Zip code</th>
<th>Zone 2</th>
<th>Zip code</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2717–2719</td>
<td>Zone 3</td>
<td>2720–2730</td>
<td>Zone 2</td>
<td>2731–2739</td>
<td>Zone 3</td>
</tr>
<tr>
<td>0200–0299</td>
<td>Zone 2</td>
<td>2740–2786</td>
<td>Zone 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000–2263</td>
<td>Zone 1</td>
<td>2787–2879</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2264–2499</td>
<td>Zone 2</td>
<td>2900–2999</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2500–2530</td>
<td>Zone 1</td>
<td>2911–2899</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2531–2554</td>
<td>Zone 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2555–2574</td>
<td>Zone 1</td>
<td>2881–2889</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2575–2647</td>
<td>Zone 2</td>
<td>2890</td>
<td>Zone 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2648</td>
<td>Zone 3</td>
<td>2891–2899</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2649–2714</td>
<td>Zone 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2715</td>
<td>Zone 3</td>
<td>2900–2999</td>
<td>Zone 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2716</td>
<td>Zone 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 — Regular parcel charges

<table>
<thead>
<tr>
<th>Weight</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Up to 250 g</td>
<td>$2.75</td>
<td>$2.75</td>
<td>$2.75</td>
</tr>
<tr>
<td>Over 250 to 500g</td>
<td>$3.75</td>
<td>$3.75</td>
<td>$3.75</td>
</tr>
<tr>
<td>Over 500 g to 1 kg</td>
<td>$5.20</td>
<td>$5.50</td>
<td>$7.40</td>
</tr>
<tr>
<td>Over 1 kg to 2 kg</td>
<td>$5.20</td>
<td>$5.80</td>
<td>$7.95</td>
</tr>
<tr>
<td>Over 2 kg to 3 kg</td>
<td>$5.20</td>
<td>$6.10</td>
<td>$8.50</td>
</tr>
<tr>
<td>Over 3 kg to 4 kg</td>
<td>$5.20</td>
<td>$6.40</td>
<td>$9.05</td>
</tr>
<tr>
<td>Over 4 kg to 5 kg</td>
<td>$5.20</td>
<td>$6.70</td>
<td>$9.60</td>
</tr>
<tr>
<td>Over 5 kg to 6 kg</td>
<td>$5.20</td>
<td>$7.00</td>
<td>$10.15</td>
</tr>
<tr>
<td>Over 6 kg to 7 kg</td>
<td>$5.20</td>
<td>$7.30</td>
<td>$10.70</td>
</tr>
<tr>
<td>Over 7 kg to 8 kg</td>
<td>$5.20</td>
<td>$7.60</td>
<td>$11.25</td>
</tr>
<tr>
<td>Over 8 kg to 9 kg</td>
<td>$5.20</td>
<td>$7.90</td>
<td>$11.80</td>
</tr>
<tr>
<td>Over 9 kg to 10 kg</td>
<td>$5.20</td>
<td>$8.20</td>
<td>$12.35</td>
</tr>
<tr>
<td>Over 10 kg to 11 kg</td>
<td>$5.20</td>
<td>$8.50</td>
<td>$12.90</td>
</tr>
<tr>
<td>Over 11 kg to 12 kg</td>
<td>$5.20</td>
<td>$8.80</td>
<td>$13.45</td>
</tr>
<tr>
<td>Over 12 kg to 13 kg</td>
<td>$5.20</td>
<td>$9.10</td>
<td>$14.00</td>
</tr>
<tr>
<td>Over 13 kg to 14 kg</td>
<td>$5.20</td>
<td>$9.40</td>
<td>$14.55</td>
</tr>
<tr>
<td>Over 14 kg to 15 kg</td>
<td>$5.20</td>
<td>$9.70</td>
<td>$15.10</td>
</tr>
<tr>
<td>Over 15 kg to 16 kg</td>
<td>$5.20</td>
<td>$10.00</td>
<td>$15.65</td>
</tr>
</tbody>
</table>

worked example

How much would it cost to send a 10.5 kg parcel to zip code 2650?

Solution

First we use Table 1 to find that zip code 2650 is in Zone 2.

Then we use Table 2 to find the charge for 10.5 kg (over 10 kg to 11 kg) in the Zone 2 column.

∴ The cost of sending the parcel is $8.50.

1 How much would it cost to send a parcel to zip code:

   a 2041 weight: 15 kg   b 2795 weight: 8.4 kg   c 2640 weight: 260 g
   d 2731 weight: 600 g   e 2983 weight: 13.8 kg   f 2530 weight: 200 g?
Chapter 2 | Revision Assignment

1 What is the value of the 1 in each of the following numerals?
   a 314  b 1608  c 21 753  d 153 246

2 Simplify:
   a 317 + 72 + 4825 + 196  
   b 2176 − 413  
   c 15 × 12  d 265 ÷ 5

3 Write these decimals as fractions.
   a 0·7  b 0·01  c 0·201  d 0·033

4 Write these fractions as decimals.
   a 3/10  b 17/100  c 45/1000  d 1 3/100

5 Simplify the following:
   a 0·5 + 0·21  
   b 4·2 + 3·17  
   c 12 + 1·5  d 7·4 + 10·05 + 3·15

6 Simplify the following:
   a 12 − 3·7  b 4·8 − 1·6  c 2·1 − 0·85  d 15·17 − 7

7 Simplify the following:
   a 6 × 0·1  b 6 × 0·01  c 0·6 × 10  d 0·6 × 0·1

8 Simplify the following:
   a 12·6 ÷ 2  b 1·04 + 2  c 5·5 + 5  d 17·24 ÷ 4

9 Simplify:
   a $1.55 + $3.75  b $15 − $7.90  c $42.20 × 7  d $24 ÷ 5

10 Simplify:
    a 3/5 + 3/5  b 37/100 + 42/100  c 15/20 − 7/20  d 19/100 − 19/100

Chapter 2 | Working Mathematically

1 Use ID Card 4 on page xvi to give the mathematical term for:
   a 1  b 2  c 3  d 4  e 5  f 6  g 7  h 8  i 9  j 13

2 Use the Arithmetic Card on page xx to:
   a multiply column A by column B  
   b add column K to column L  

3 A girl cuts six lengths of cloth from a roll 20 m long. If each length is 2·4 m, how much cloth remains?

4 A machine caps bottles at a speed of one bottle every 2 seconds. How many bottles does the machine cap in 4 hours?

5 How could you estimate how many households have a phone in your capital city?

6 You are running a basketball competition in which six teams are playing. Work out the draw for the competition if each team has to play each other team once only.

7 A suburban railway line has 10 stations. How many different tickets would be needed by the ticket office to cover any trip from one station to another?
Dave depended on the water stored in his tank to meet his everyday needs. This graph shows the depth of water in the tank from 8 am to 8 pm.

During the day it rained, Dave had a bath and Dave’s son Brian left the tap running.

a. Between what two times did it rain?
b. What was the highest level of water in the tank during this 12-hour period?
c. When did Brian leave the tap on?
d. When was the tank empty?
e. How much water was in the tank at 2:30 pm?